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2-8-12 2.7 Chain Rule

Recall

$D_x[f(x)]^n = n[f(x)]^{n-1} D_x(f(x))$

$D_x(\sin(f(x)) = \cos(f(x)) D_x f(x))$

$D_x(\cos(f(x)) = -\sin(f(x)) D_x(f(x))$

$D_x(\tan(f(x)) = \sec^2(f(x) D_x(f(x))$

$D_x(\cot(f(x)) = -\csc^2(f(x) D_x(f(x))$

$D_x(\sec f(x)) = \sec(f(x)) \tan(f(x)) D_x(f(x))$

$D_x(\csc f(x)) = -\csc(f(x)) \cot(f(x)) D_x(f(x$

$D_x(\sqrt{f(x)}) = \frac{1}{2\sqrt{f(x)}} D_x(f(x))$

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ex1 Differentiate

a) $y=(x^2-3x+4)^5$

$y' = 5(x^2-3x+4)^4(2x-3)$

$= 5(2x-3)(x^2-3x+4)^4$

b) $y=(x^2-3)^4(2x+1)^9$

$y' = (x^2-3)^4\,9(2x+1)^8(2) + (2x+1)^9\,4(x^2-3)^3(2x)$

$= 18(x^2-3)^4(2x+1)^8 + 8x(x^2-3)^3(2x+1)^9$

$= 2(x^2-3)^3(2x+1)^8\left(9(x^2-3)+4x(2x+1)\right)$

$= 2(x^2-3)^3(2x+1)^8(9x^2-27+8x^2+4x)$

$= 2(x^2-3)^3(2x+1)^8(17x^2+4x-27)$

c) $y=5\left(\frac{2x-3}{x^2+1}\right)^6$

$y' = 5\left[6\left(\frac{2x-3}{x^2+1}\right)^5 D_x\left(\frac{2x-3}{x^2+1}\right)\right]$

$y' = 30\left(\frac{2x-3}{x^2+1}\right)^5\left(\frac{(x^2+1)(2)-(2x-3)(2x)}{(x^2+1)^2}\right)$

$= 30\left(\frac{2x-3}{x^2+1}\right)^5\left(\frac{2x^2+2-4x^2+6x}{(x^2+1)^2}\right)$

$= 30\left(\frac{2x-3}{x^2+1}\right)^5\left(\frac{-2x^2+6x+2}{(x^2+1)^2}\right) = \frac{30(2x-3)^5(2+6x-2x^2)}{(x^2+1)^7}$

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(d) $y = \sin^7(x) = [\sin(x)]^7$

$y' = 7[\sin(x)]^6 \cos x$

$= 7 \sin^6 x \cos x$

(e) $y = \sin x^7 = \sin(x^7)$

$y' = \cos(x^7) \cdot (7x^6)$

$= 7x^6 \cos x^7$

(f) $y = \csc 7x - \tan x^2 + 5\sec(4)$

$y' = -\csc 7x \cot 7x\,(7) - \sec^2(x^2)(2x) + 0$

$= -7\csc 7x \cot 7x - 2x\sec^2(x^2)$

(g) $p = \cot^3(q^2+2) = [\cot(q^2+2)]^3$

$\frac{dp}{dq} = 3[\cot(q^2+2)]^2 \, D_q(\cot(q^2+2))$

$= 3[\cot(q^2+2)]^2(-\csc^2(q^2+2))(2q)$

$= -6q \cot^2(q^2+2) \csc^2(q^2+2)$

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(h) $y = (4+\sqrt{6x-1})^5 = (4+(6x-1)^{1/2})^5$

$y' = 5(4+\sqrt{6x-1})^4\left(\frac{1}{\cancel{2}\sqrt{6x-1}}\right)(\overset{3}{\cancel{6}})$

$y' = \frac{15(4+\sqrt{6x-1})^4}{\sqrt{6x-1}}$

(i) $y = \csc x \sin x = \frac{1}{\sin x}\sin x = 1$

$y' = 0$

(j) $y = \csc \sin x = \csc(\sin x)$

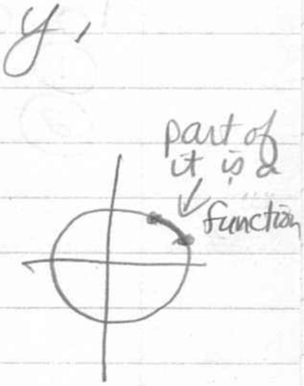
$y' = -\csc(\sin x)\cot(\sin x)(\cos x)$

$= -\cos x \csc(\sin x)\cot(\sin x)$

2.8 Implicit Differentiation

So far we have always been given y explicitly as a function of x. (that is $y = f(x)$).
Often we may just have an equation that relates x and y.

e.g. $x^2 + y^2 = 25$



Such an equation is said to define y implicitly as a function of x.

In the case of $x^2+y^2=25$ we can find the function implicit in the equation

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$x^2+y^2=25$

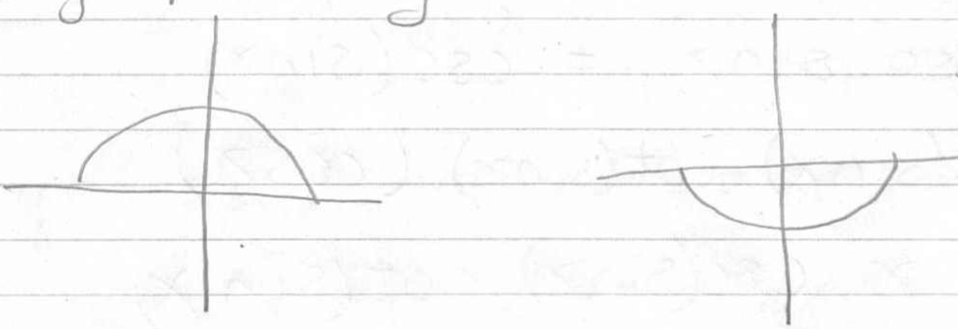
$y^2=25-x^2$

$y=\pm\sqrt{25-x^2}$

$y=\sqrt{25-x^2}$ $y=-\sqrt{25-x^2}$

these are the functions implicit in the equation

graphically



Here you could solve for y and pick the equation.

$e^x+y^2=\sin(xy)$

Here you cannot solve for y.

To differentiate implicitly we
1. Assume y is a function of x
2. Differentiate both sides of the equation.

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ex 1 Assume y is a function of x and differentiate.

(a) $D_x(y^4) =$

$D_x(f(x))^4 = 4\,(f(x))^3 \cdot f'(x)$

$D_x(y^4) = 4y^3\,y'$ or $4y^3\frac{dy}{dx}$

(b) $D_x(x^2y^2) =$

$= x^2 D_x y^2 + y^2 D_x(x^2)$

$= x^2\,2y\,y' + y^2\,2x$

Note: $D_x(x) = 1$

$D_x(y) = y'$

(c) $D_x(x^3 \sin(y))$

$= x^3(\cos(y)\,y') + \sin y\,(3x^2)$

$= x^3\cos(y)\,y' + 3x^2\sin y$

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ex 2 Find $\frac{dy}{dx}$ if

$$x^2+y^3=3xy^2-4$$

$$D_x(x^2+y^3)=D_x(3xy^2-4)$$

$$2x+3y^2y'=3x(2y\,y')+y^2(3)$$

$$2x+3y^2y'=6xy\,y'+3y^2$$

$$3y^2y'-6xy\,y'=3y^2-2x$$

$$y'(3y^2-6xy)=3y^2-2x$$

$$y'=\frac{3y^2-2x}{3y^2-6xy}$$

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ex 3 Find $\frac{da}{db}$ if $a^2 + 3ab + b^3 = 17$

$$D_b(a^2 + 3ab + b^3) = D_b(17)$$

$$2a\,a' + 3a + b(3a') + 3b^2 = 0$$

$$2a\,a' + 3b\,a' = -3b^2 - 3a$$

$$a' = \frac{-3b^2 - 3a}{2a + 3b}$$

ex4
Find the equations of the lines tangent and normal to $x^3 + 3y^2 + 7y = 4$ at $(2,-1)$

$x^3 + 3y^2 + 7y = 4$ Point $(2,-1)$

$D_x(x^3 + 3y^2 + 7y) = D_x 4$ slope $= \frac{dy}{dx}$

$$3x^2 + 6y\,y' + 7y' = 0$$

$x = 2,\ y = -1 \quad y' = m$

$$3(4) - 6m + 7m = 0$$

$$m = -12$$

tangent

$y + 1 = -12(x-2)$

normal

$y + 1 = \frac{1}{12}(x-2)$

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$y = \sqrt[n]{x^m} = x^{\frac{m}{n}}$

$y' = \frac{m}{n} x^{\frac{m}{n}-1}$